

Due Friday, February 28, 2025.

Write your homework *neatly, in pencil*, on blank white  $8\frac{1}{2} \times 11$  printer paper. Always *write the problem*, or at least enough of it so that your work is readable. If the problem involves a function, write the function. If the problem involves an equation, write the equation. Use words, and when appropriate, *write in sentences*.

**Definition 1.** Define the *natural logarithm* to be the function

$$\log : (0, \infty) \rightarrow \mathbb{R} \quad \text{given by} \quad \log(x) = \int_1^x \frac{1}{t} dt.$$

We have shown that  $\log$  is bijective.

**Definition 2.** Define the *natural exponential function* to be the inverse of the natural logarithm. Thus

$$\exp : \mathbb{R} \rightarrow (0, \infty) \quad \text{such that} \quad \exp(x) = y \Leftrightarrow x = \log(y).$$

Define the number  $e$  by

$$e = \exp(1).$$

For  $a \in (0, \infty) \setminus \{1\}$  and  $x \in \mathbb{R}$ , define

$$a^x = \exp(x \log(a)) \quad \text{so that} \quad e^x = \exp(x).$$

We have shown that  $\frac{d}{dx} \exp(x) = \exp(x)$ , so  $\int \exp(x) dx = \exp(x) + C$ .

**Problem 1** (Thomas §7.3 # 9). Solve for  $y$  where

$$\ln(y - 1) - \ln(2) = x + \ln x.$$

**Problem 2** (Thomas §7.3 # 21). Find  $\frac{dy}{dx}$  where

$$y = xe^x - e^x.$$

**Problem 3** (Thomas §7.3 # 35). Find  $\frac{dy}{dx}$  where

$$y = \int_0^{\ln x} \sin(e^t) dt.$$

**Problem 4** (Thomas §7.3 # 47). Evaluate the definite integral

$$\int_{\ln 4}^{\ln 9} e^{x/2} dx.$$

**Problem 5** (Thomas §7.3 # 53). Evaluate the indefinite integral

$$\int \frac{e^{1/x}}{x^2} dx.$$

**Problem 6** (Thomas §3.5 # 36). Find  $\frac{dr}{d\theta}$  where

$$r = \sec \sqrt{\theta} \tan \left( \frac{1}{\theta} \right).$$

**Problem 7** (Thomas §3.8 # 9). Let

$$f(x) = \sqrt[3]{x}.$$

Find the linearization of  $f$  at  $x = 8$ , and use it to estimate  $\sqrt[3]{8.5}$ .

**Problem 8** (Thomas §7.3 # 67). Let

$$f(x) = e^x - 2x.$$

Find the absolute maximum and minimum values of  $f$  on  $[0, 1]$ .

**Problem 9.** Let  $R$  be the region bounded by  $x = 0$ ,  $x = \ln(5)$ ,  $y = 0$ , and  $y = e^x$ . Find the volume of the solid obtained by revolving  $R$  about the  $x$ -axis

**Problem 10.** Let  $R$  be the region bounded by  $x = 0$ ,  $x = \ln(5)$ ,  $y = 0$ , and  $y = e^x$ . Find the volume of the solid obtained by revolving  $R$  about the  $y$ -axis